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Simplified mathematical model for calculating global warming through anthropogenic CO_2

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ABSTRACT

A simplified mathematical model has been developed to understand the effect of carbon dioxide on the mechanism of global warming. The heat transfer in the atmosphere, in particular the radiation, can be described by known relations in thermal engineering. Here, the radiation exchange among the gas which contains water vapor and carbon dioxide and the Earth's surface as well as the clouds is considered. The emissivity of the gases are a function of temperature, gas concentration and beam length of the atmospheric layer defined through the model. The model is validated with the known average temperature of the Earth. The emissivity of the clouds acts as an adjustment parameter. The temperature of the Earth increases significantly with the CO_2 concentration. While doubling the concentration of CO_2, the temperature of the Earth increases by 0.4 K.

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1. Introduction

Global warming is currently a widely discussed topic on which opinions greatly vary. On one side, there are climatologists, who are convinced that global warming will lead to a climatological catastrophe. On the other side, few references say that the temperature change is caused by natural climate fluctuations. Numerous multilayer models have already been developed to calculate the effects of human influence on the climate [e.g. Refs. [1–3]]. These models are often very complex and difficult to comprehend due to the elusive discussion about the relation between the properties of the atmospheric layers and the radiative behavior of the involved gases such as water vapor and carbon dioxide.

Therefore, a simplified mathematical model was established in order to show how anthropogenic carbon dioxide affects the global temperature of the Earth. With the model, the mechanism of global warming caused by CO_2 can be made understandable. Especially, the focus is on the determination of the radiative gas properties using the temperature, gas concentration and beam length of the atmospheric layers defined through the model.

The description of complex processes in industry by using simplified mathematical models is a common practice in engineering. That such models are limited in their accuracy is accepted in this context. Simplified models are necessary to make the principal effect of the influences comprehensible and to find a solution quickly. Such rapidly found solutions are only approximate solutions. When there is more time to solve the problem, the apparatus is simulated with complex, multidimensional mathematical models. Here, the highest possible accuracy of the results will be expected. Even complex mathematical models are only accepted in the industrial practice if they are validated with experimental results. If applicable, the models are adjusted to realistic conditions by choosing different parameters, for example, the heat transfer coefficient or the reaction coefficient.

An analogue approach is taken for the simulation of the influence of anthropogenic CO_2 on global warming. For describing the mechanism of the greenhouse effect, it is sufficient to consider the Earth with an uniform temperature. Therefore, no distinction was made between land and oceans. A calculation of the local temperature distribution on the Earth is not the purpose of the model. The modeling of the greenhouse effect in the atmosphere is similar to the modeling of thermal processes in industrial furnaces. There are similar heat transfer mechanisms by radiation in the atmosphere. There is also a gas consisting of H_2O and CO_2 (the combustion gas) in the radiation exchange between bodies of different temperature.

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Nomenclature

Symbol

R	Resistance
R_{H_2O}	Gas constant of water e.g. J kg^{-1} K^{-1}
R_I	Heat exchange resistance between Earth and clouds
T	temperature °C, K
c	Concentration, mole fraction
ė	Emissive power W m^{-2}
g	Gravity m s^{-2}
ḣ	Brightness W m^{-2}
k	Number of resistances
p	Total pressure, partial pressure Pa
q̇	Heat flux W m^{-2}
s	Beam length m
z	Height, altitude m
Δh	Enthalpy change e.g. J kg^{-1}
ε	Emissivity
ϕ	View factor
φ	Relative humidity
σ	Stefan-Boltzmann constant W m^{-2} K^{-4}

Index

0	Standard conditions
∞	Space
C	Clouds
CON	Convection
E	Earth
G	Gas
I	Heat exchange resistance between Earth and clouds e.g.
RAD	Radiation
SO	Solar
VAP	Vaporization
b	Black body
s	saturated

The reasons for the historical increase in temperature are various. The effect of CO_2 and H_2O play an important role, but there are side effects, other greenhouse gases e.g. methane and further reasons not discussed here. Between 1860 and 1990 was the global warming 1 K. CO_2 contributed only 0.4 K. A validation with the historical data requires a complex model including more parameters [e.g. Ref. [4]]. Therefore, the model is validated with the known average temperature of the Earth.

2. Global energy balance Earth-atmosphere

2.1. Model for radiation and energy flows through the atmosphere

Radiation from the Sun is the only source of energy for the Earth. The heat flux from the Sun can be measured. At the upper boundary of the atmosphere, the Earth obtains a heat flow rate of 1368 W m^{-2}. This parameter is named the extra-terrestrial solar constant. Since the cross sectional area of the Earth is only a quarter of its surface area, an average of 342 W is allotted to each square meter. The energy balance of the Earth and its atmosphere presented by Kiehl and Trenberth, 1997 [5] is shown in Fig. 1.

About 30% of the shortwave radiation from the Sun is reflected back into Space by the atmosphere and the Earth's surface. The ratio of the radiation reflected back by the Earth to the incident radiation is called albedo. Consequently, the heat flow rate is reduced to 235 W m^{-2}. The clouds and the atmosphere absorb about 20% of the radiation and the remaining 50% is absorbed on the Earth's surface (land, ocean). In thermal equilibrium, the energy supplied to the Earth must also be released. This occurs through the heat emission of the Earth's surface, which amounts to 390 W m^{-2}, under the assumption that the Earth is an ideal grey body. By comparing this heat emission with the Sun's radiation, it becomes clear that the radiation of the Earth's surface is higher. A part of this long wave radiation reaches Space directly through the atmospheric window (40 W m^{-2}); the other part is absorbed by the atmosphere, clouds and greenhouse gases. Table 1 gives an overview of the most important greenhouse gases and their concentration in the atmosphere [6]. As shown in the table, H_2O and CO_2 are the most present gases in the atmosphere. The concentration of methane and nitrous oxide is substantially smaller.

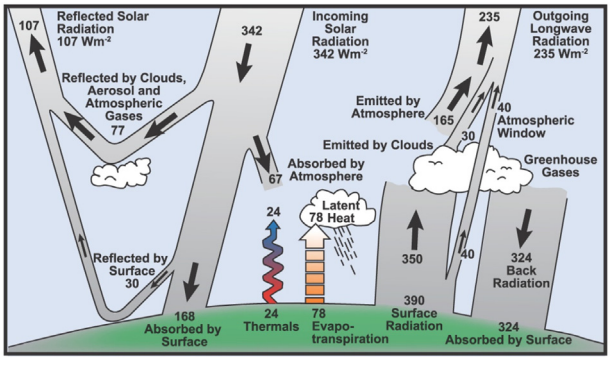

Fig. 1. Heat fluxes in the atmosphere [courtesy: Kiehl and Trenberth, 1997 [5]].

The greenhouse gases and clouds radiate 195 W m^{-2} into Space, as well as yield a counter radiation of 324 W m^{-2} to the Earth's surface. This causes a positive energy balance for the Earth's surface and in the same proportion, a negative balance for the atmosphere. In order for the radiation balance to be equal to zero, equilibrium between the Earth's surface and the atmosphere must be obtained. Therefore, the two heat fluxes, convection (24 W m^{-2}) and evaporation (78 W m^{-2}), are included in the energy balance. This description of the heat fluxes is found in many fields of climatology. In the following, this heat flow with mathematical relationships will be described in order to make the influence of CO_2 clear.

2.2. Mathematical modeling

For the mathematical description of heat fluxes, the back radiation will not be described. Rather, the net heat fluxes exchanged according to the Stefan–Boltzmann law are considered as is usual in thermal engineering. The effective heat fluxes are summarized in Fig. 2. About 50% of the Earth is covered with clouds [7,8]. The clouds as well as dust, aerosols, and a shortwave band of H_2O in the atmosphere act as a radiation shield, the total radiation at the Earth's surface is reduced. Consequently, the heat flow rate is

Table 1
Greenhouse gases [6].

Gas	Concentration (2014)
Water vapor	variable 0–2%
Carbon dioxide	397 ppm
Methane	1815 ppb
Nitrous Oxide	326 ppb

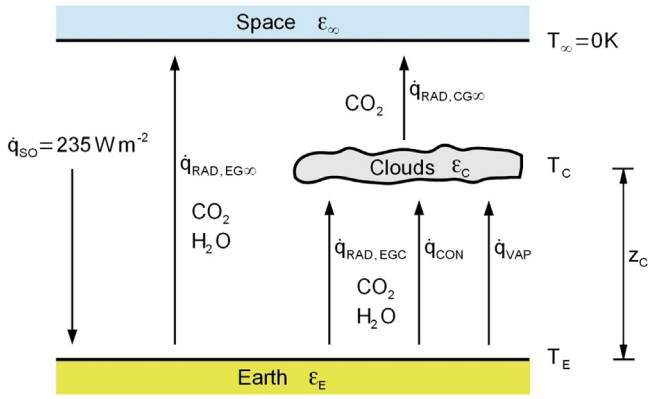

Fig. 2. Simplified description of the heat fluxes in the atmosphere.

reduced to 235 W m^{-2} as in Fig. 1 presented. In thermal equilibrium the solar radiation that reaches the Earth's surface must be released into Space. One part of the heat is radiated through the cloud-free sky directly into Space. The other part is transferred by radiation, convection and evaporation to the clouds. Therefore, the total heat flux can be described using

$$\dot{q}_{SO} = \dot{q}_{RAD,EG\infty} + \dot{q}_{RAD,CG\infty} \tag{1}$$

with

$$\dot{q}_{RAD,CG\infty} = \dot{q}_{RAD,EGC} + \dot{q}_{CON} + \dot{q}_{VAP}. \tag{2}$$

Subsequently, the clouds radiate the transferred heat fluxes on its upper surface into Space. Above the clouds, almost no H_2O exists, because it is condensed into clouds. Consequently, only CO_2 exists as a radiating gas.

Neglecting other greenhouse gases, such as methane and nitrous oxide, because of their low concentrations allows to discuss the effect of CO_2 and H_2O in the atmosphere on the global warming separately. Absorption capacity, concentration and residence time of gases are used to evaluate the so called global warming potential. Indeed, methane and nitrous oxide have high global warming potential as well. The reason is because of the long residence time in the atmosphere though. So, the heat fluxes in the atmosphere are not significantly influenced.

Also, it is simplified that the heat is transported without resistance through the clouds and the clouds thus have a uniform temperature.

The following equations are valid for the three heat fluxes by radiation:

$$\dot{q}_{RAD,EG\infty} = \varepsilon_{EG\infty} \cdot \sigma \cdot \left(T_E^4 - T_\infty^4\right), \tag{3}$$

$$\dot{q}_{RAD,EGC} = \varepsilon_{EGC} \cdot \sigma \cdot \left(T_E^4 - T_C^4\right), \tag{4}$$

$$\dot{q}_{RAD,CG\infty} = \varepsilon_{CG\infty} \cdot \sigma \cdot \left(T_C^4 - T_\infty^4\right), \tag{5}$$

where T_E, T_C, $T_\infty$are the temperatures of the Earth, the clouds and of the Space, $\varepsilon_{EG\infty}$, ε_{EGC}, $\varepsilon_{CG\infty}$ are the effective emissivities and σ is the Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}).

On Earth, the average annual precipitation is 1000 mm m^{-2} a^{-1}. This water quantity evaporates on the surface and condenses into clouds. The vaporization enthalpy of water of 2500 kJ kg^{-1} leads to a specific heat flux of $\dot{q}_{VAP} = 78$ W m^{-2}. The specific heat flux due to convection is according to Fig. 1 $\dot{q}_{CON} = 24$ W m^{-2}, which corresponds to an average heat transfer coefficient of about 2 W m^{-2} K^{-1}. This is a typical value for free convection of horizontal plates. The calculation of the radiative heat fluxes and the temperature of the Earth and the clouds will be discussed in the following chapter.

2.3. Heat transfer by radiation

Using the network method the radiation exchange between two surfaces and a radiant gas can be calculated [9–11]. The network analogy for the Earth, gas, and cloud system is shown in Fig. 3.

For the total heat flow resulting between the vertices, Earth and clouds, the following equation is defined.

$$\dot{q}_{RAD,EGC} = \frac{\dot{e}_{bE} - \dot{e}_{bC}}{R_{total}}, \tag{6}$$

where $\dot{e}_{bE}$, $\dot{e}_{bC}$ are the emissive power of the Earth and the clouds as black bodies and R_{total} the total resistance between Earth and clouds. To calculate the total resistance, Kirchhoff's Law from electrical engineering is used. If k resistances are connected in series the individual resistances are summated as $R_{total} = \sum_k R_k$. For a parallel circuit of k resistances, the reciprocals of the resistances are summated as $\frac{1}{R_{total}} = \sum_k \frac{1}{R_k}$.

In order to calculate the total resistance of the network shown in Fig. 3, first the partial resistance R_I is formed, which is between the brightness points $\dot{h}_C$ and $\dot{h}_E$. The resistances R_{GC} and R_{EG} are connected in series and lie parallel to the resistance R_{EC}. Therefore, the partial resistance R_I is given by Eq. (7)

$$R_I = \frac{1}{\frac{1}{R_{EC}} + \frac{1}{R_{GC}+R_{EG}}}. \tag{7}$$

The total heat flux $\dot{q}_{RAD,EGC}$ has to overcome the resistances R_E, R_I, and R_C between Earth, gas and clouds. Since the three resistances are connected in series, the total resistance of the system is calculated through Eq. (8)

$$R_{total} = R_E + R_I + R_C. \tag{8}$$

Eq. (7) is substituted into Eq. (8), resulting in the following total resistance:

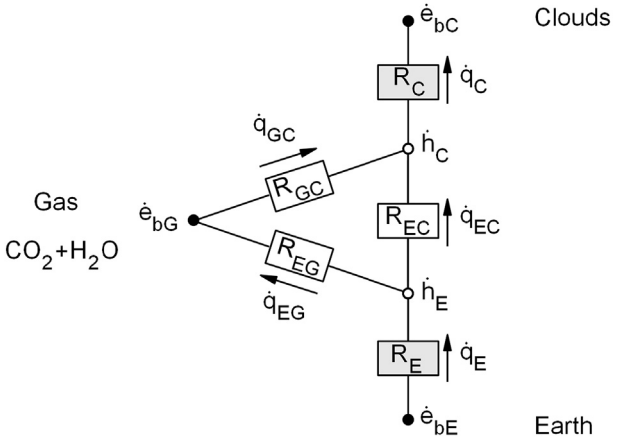

Fig. 3. Equivalent circuit diagram for the radiation exchange.

$$R_{total} = R_E + \frac{1}{\frac{1}{R_{EC}} + \frac{1}{R_{GC}+R_{EG}}} + R_C. \quad (9)$$

The surface resistances are given assuming grey radiation as:

$$R_E = \frac{1-\varepsilon_E}{\varepsilon_E}, \quad R_C = \frac{1-\varepsilon_C}{\varepsilon_C}, \quad (10)$$

where ε_E is the emissivity of the Earth and ε_C is the emissivity of the clouds. The radiation exchange resistances can be calculated as follows:

$$R_{EC} = \frac{1}{\phi_{EC}\cdot\tau_{GEC}}, \quad R_{EG} = \frac{1}{\varepsilon_{EG}}, \quad R_{GC} = \frac{1}{\varepsilon_{GC}}. \quad (11)$$

Here τ_{GEC} is the transmissivity of the gas, ε_{EG} and ε_{GC} the effective emissivities between gas and Earth or clouds respectively. Assuming grey gas behavior the transmissivity can be expressed by the emissivity $\tau_{GEC} = 1-\varepsilon_{GEC}$. Since the distance of the clouds is small compared to the Earth curvature, the view factor is determined $\phi_{EC} = 1$. Considering the encountered assumptions, the total resistance can be calculated as follows:

$$R_{total} = \frac{2}{1-\varepsilon_{GEC}} + \frac{1}{\varepsilon_E} + \frac{1}{\varepsilon_C} - 2. \quad (12)$$

The emissive black power is given by

$$\dot{e}_{bE} = \sigma\cdot T_E^4, \quad \dot{e}_{bC} = \sigma\cdot T_C^4. \quad (13)$$

The heat flux can be expressed with the effective emissivity

$$\dot{q}_{RAD,EGC} = \varepsilon_{EGC}\cdot\sigma\cdot\left(T_E^4 - T_C^4\right) \quad (14)$$

for which the following is obtained from Eq. (12)

$$\varepsilon_{EGC} = \frac{1}{\frac{2}{2-\varepsilon_{GEC}} + \frac{1}{\varepsilon_E} + \frac{1}{\varepsilon_C} - 2}. \quad (15)$$

For the radiation of the clouds into Space and of the Earth into Space, the same mechanism can be employed. Since the Space is a black body ($\varepsilon_\infty = 1$), it follows:

$$\dot{q}_{RAD,CG\infty} = \varepsilon_{CG\infty}\cdot\sigma\cdot T_C^4 \text{ and } \dot{q}_{RAD,EG\infty} = \varepsilon_{EG\infty}\cdot\sigma\cdot T_E^4 \quad (16)$$

with

$$\varepsilon_{CG\infty} = \frac{1}{\frac{2}{2-\varepsilon_{GC\infty}} + \frac{1}{\varepsilon_C} - 1} \text{ and } \varepsilon_{EG\infty} = \frac{1}{\frac{2}{2-\varepsilon_{GE\infty}} + \frac{1}{\varepsilon_E} - 1}. \quad (17)$$

Because both surface emissivities are close to one the approximations are introduced

$$\dot{q}_{RAD,EGC} \approx \varepsilon_E\cdot\varepsilon_C\cdot\left(1-\frac{1}{2}\varepsilon_{GEC}\right)\cdot\sigma\cdot\left(T_E^4 - T_C^4\right), \quad (18a)$$

$$\dot{q}_{RAD,CG\infty} \approx \varepsilon_C\cdot\left(1-\frac{1}{2}\varepsilon_{GC\infty}\right)\cdot\sigma\cdot T_C^4, \quad (18b)$$

$$\dot{q}_{RAD,EG\infty} \approx \varepsilon_E\cdot\left(1-\frac{1}{2}\varepsilon_{GE\infty}\right)\cdot\sigma\cdot T_E^4. \quad (19)$$

Therefore, the effect of the gas radiation can be demonstrated very clearly. The higher the emissivity of the gas is, the lower the effective emissivity is. Thus, the gas acts as a radiation shield, which impedes the radiation. The effective emissivity cannot be less than 0.5, so that even with an optically thick gas heat is transferred through. In this study the heat flux from the Sun is constant. Therefore, the higher resistance due to gas radiation has to compensate by raising the temperature of the Earth.

In the following section the required emissivities of the gases are calculated.

2.4. Determination of emissivity of gases

2.4.1. Optical thickness

The emissivity of gases depends on the product of partial pressure and beam length as well as on the temperature. In the atmosphere, the temperature and the partial pressure decrease with higher altitude, so that suitable mean values must be determined.

In Fig. 4 the temperature in the atmosphere is shown. The temperature decreases linearly by 0.6 K per 100 m, which can be theoretically derived from the first law of thermodynamics and the equation of state for ideal gas behavior [12]. Using the relation of Clausius-Clapeyron to calculate the saturated pressure p_s of water vapor at a particular temperature T as follows

$$p_s = p_0\cdot\exp\left(-\frac{\Delta h_{VAP,0}}{R_{H2O}}\cdot\left[\frac{1}{T}-\frac{1}{T_0}\right]\right), \quad (20)$$

the partial pressure of water vapor can be written as

$$p_{H2O} = \varphi\cdot p_s \quad (21)$$

where p_0 is the saturated pressure, $\Delta h_{VAP,0}$ the evaporation enthalpy of water at standard conditions depending on T_0, R_{H2O} is the gas constant of water (464 J kg^{-1} K^{-1}) and φ is the relative humidity of the atmosphere. The figure also shows the associated decrease of the partial pressure of water vapor, wherein the average humidity of the atmosphere is assumed to be $\varphi = 80\%$.

The integration over the height leads to the mean product of partial pressure and beam length

$$(p_{H_2O}\cdot s)_{EC} = \int_0^{z_C} p_{H_2O}(z)\cdot dz, \quad (p_{H_2O}\cdot s)_{E\infty} = \int_0^{\infty} p_{H_2O}(z)\cdot dz. \quad (22)$$

For the emissivity below the clouds the product of partial pressure and beam length is integrated from the surface to the height of the clouds z_c, which is approximately on average at 3 km. For the emissivity of the cloud-free sky an integration to infinity is needed. The value of the integral, the optical thickness, is shown in Fig. 5. The final accumulation is reached at a height of about 8 km.

For determining the optical thickness of the carbon dioxide, it must be taken into account that the concentration c is constant with the height, but the total pressure deceases according to the barometric formula:

$$p(z) = p_0\cdot\exp\left[-\frac{g\cdot z}{R_{air}\cdot T_{air}}\right], \quad (22a)$$

where $p_0 = 1$ bar is the pressure at the ground, R_{air} is the gas constant of air (287 J kg^{-1} K^{-1}), g is the gravity constant, T_{air} is the mean temperature of the air and z is the coordinate of the height. With $p_{co_2} = p\cdot c_{co_2}$ the optical thickness follows

$$p_{CO_2}\cdot s = c_{CO_2}\cdot\int_0^z p(z)\cdot dz = c_{CO_2}\cdot\int_0^z p_0\cdot\exp\left(-\frac{z\cdot g}{R_{air}\cdot T_{air}}\right)\cdot dz. \quad (23)$$

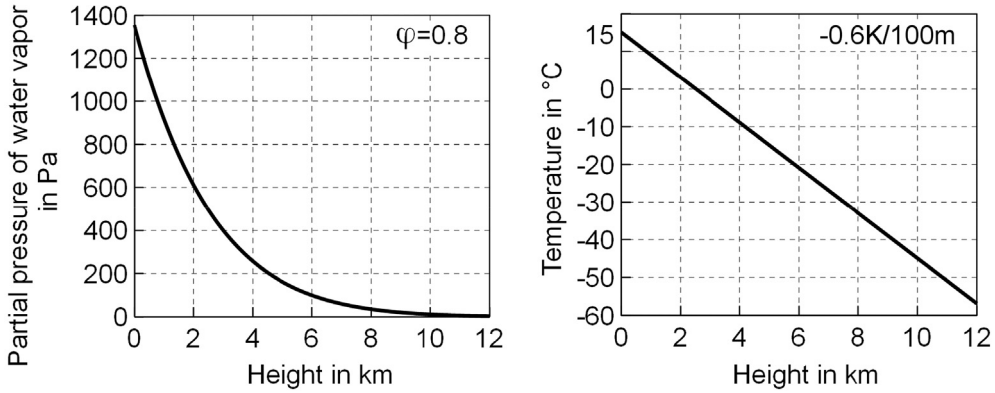

Fig. 4. Decrease of temperature and partial pressure of water vapor in the Atmosphere with height.

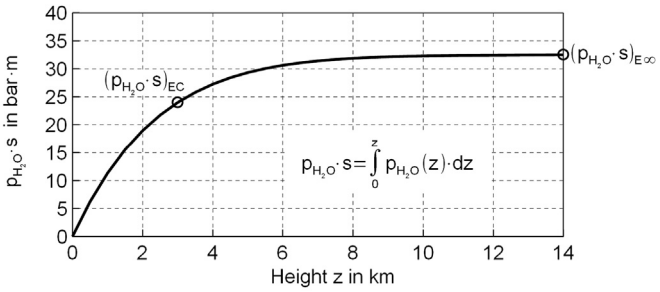

Fig. 5. Optical thickness of water vapor in the atmosphere.

The results are shown in Fig. 6. The concentration of 355 ppm is used, which was measured in the reference year 1990 for the assessment of CO_2 reduction. The optical thickness for the gas between the Earth and the clouds is 0.90 bar·m, for a cloud-free sky and Space it is 2.72 bar·m and for the gas between the clouds and Space it is 1.43 bar·m. The final accumulation is reached at a height of about 20 km. Only the radiation behavior below this level influences the Earth's temperature.

In Table 2, the products of the partial pressure and the beam length of the individual gases are presented. For the relevant average temperatures of the atmosphere, values are selected, which belong to the half of the maximum value of the optical thickness. These temperatures are 7 °C below the clouds, −55 °C above the clouds and −15 °C for the cloud-free atmosphere.

The temperature above the clouds is often displayed in airplanes as the outside temperature. As shown in Fig. 6, the flying height of 11 km lies approximately in the average range of the optical thickness.

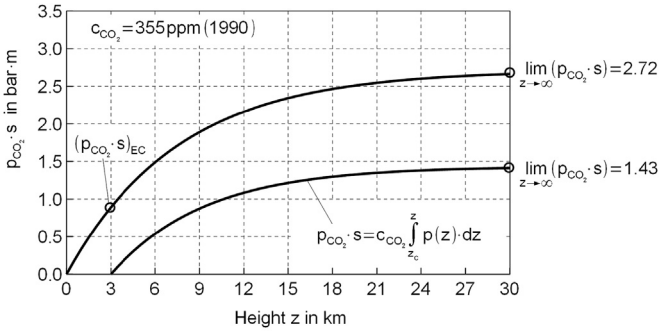

Fig. 6. Optical thickness of carbon dioxide in the atmosphere.

Table 2
Optical thickness of the individual gases.

Gas i	$p_i·s$ = bar·m		
	Below the clouds T = 7 °C	above the clouds T = −55 °C	Cloud-free sky T = −15 °C
H_2O	23.8	–	32.3
CO_2	0.90	1.43	2.72

2.4.2. *Emissivity of H_2O and CO_2*

With dependence on partial pressure, beam length and temperature, the emissivity of both H_2O and CO_2 can be found in the figures and diagrams of heat transfer books [10,11]. Using these graphical solutions is the most practical method. But in all heat transfer books the emissivity is only specified above 20 °C. Especially temperatures below 20 °C are relevant to describe the effect of greenhouse gases in the atmosphere. Therefore, reasonable approximations are necessary.

The emissivity of water vapor is shown in Fig. 7. The values used are taken from Ref. [7]. The authors explain the benchmark measuring values of the spectral emissivities and compare various numerical calculation methods of the overall emissivity in the entire wavelength range. It is obviously, that the emissivity at large values of the product of pressure and beam length, at about more than 5 bar · m, is nearly independent of temperature [13–15]. In literature, no emissivities could be found for optical thicknesses above 10 bar m. So, an extrapolation is used to assume the

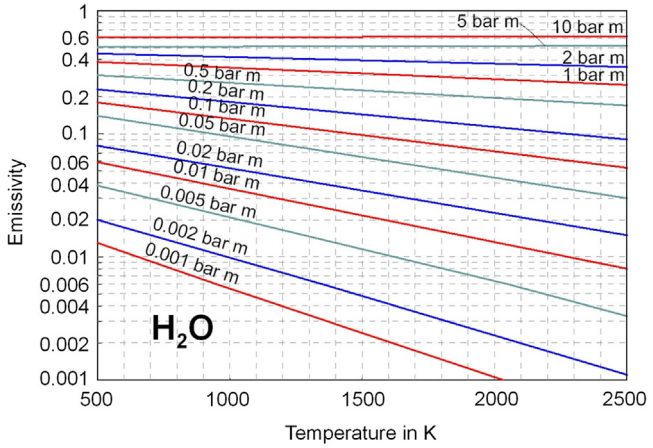

Fig. 7. Emissivity of water vapor [10,15].

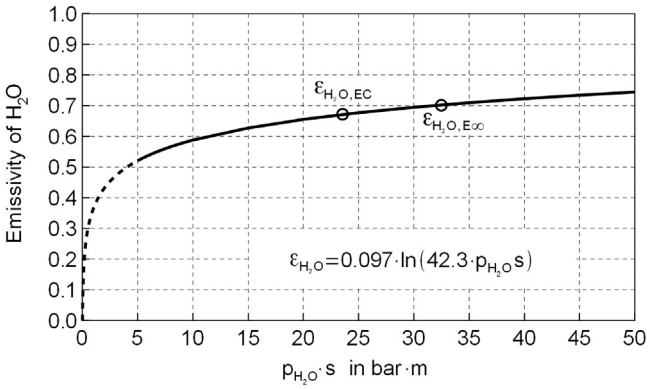

Fig. 8. Emissivity of water vapor in the atmosphere.

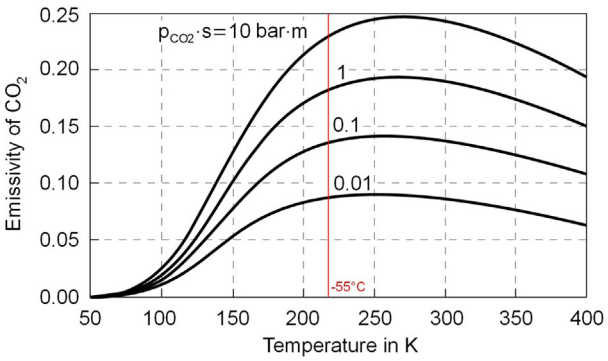

Fig. 10. Emissivity of carbon dioxide at low temperature [14].

emissivity of water vapor within the relevant region of the atmosphere, as presented in Table 2. Fig. 8 shows the emissivity of water vapor as a function of the optical thickness. The approximation used is given within the figure as well. For the relevant values of the atmosphere the graph stays relatively flat.

The emissivity of carbon dioxide is shown in Fig. 9. The values are taken again from Lallemant et al. [15]. At short-wave radiation and high temperature the emissivity goes to zero, because carbon dioxide has no absorption bands less than 2 μm. The largest absorption band of carbon dioxide is in the range of 12–18 μm. Only this band is significant for the radiation in the atmosphere. Therefore, the emissivity shows a maximum at a temperature of about 250 K [10,16]. At wavelengths above 18 μm carbon dioxide has no bands, therefore the emissivity goes to zero at temperatures below 70 K and extremely long-wave radiation [9].

Especially at temperature below 20 °C, Lorenz [16] determined the emissivity considering only the relevant 15 μm absorption band of carbon dioxide. This author based her calculation on various models [e.g. Refs. [17,18]]. It is clearly visible in Fig. 10, that the emissivity not only rises with the concentration, but also with an increase of temperature. The increase of the temperature in the atmosphere results in an increase of the emissivity, even with a constant CO_2 concentration.

The emissivity of the carbon dioxide as a function of the optical thickness is shown in Fig. 11. The illustration shows that all three emissivities are situated in the flat range of the curve. Obviously, the emissivity would increase if the optical thickness increases further due to an increase of a concentration of carbon dioxide.

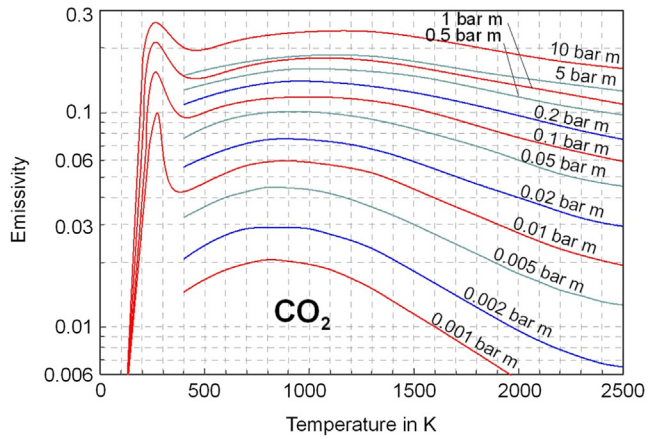

Fig. 9. Emissivity of carbon dioxide [10].

The gas below the clouds and the gas of cloud-free sky consist of H_2O and CO_2. The emissivity of the mixture is given as:

$$\varepsilon_G = \varepsilon_{CO_2} + \varepsilon_{H_2O} - \varepsilon_{CO_2} \cdot \varepsilon_{H_2O}. \quad (24)$$

Fig. 12 demonstrates that the gas emissivity insignificantly increases for CO_2 concentrations above the current value of 397 ppm. However, the emissivity of the gas above the clouds still has to be considered, because the emissivity below the clouds is dominated by H_2O.

2.5. Determination of emissivity of gases

By using the previously given equations, the heat fluxes in the atmosphere due to radiation can be calculated. The mean emissivity of the Earth is 0.95 [13,14]. The average value of the emissivity of the clouds for the entire atmosphere is unknown, so the temperature of the Earth cannot be calculated with the model presented here. The aim of the model is not to calculate the temperature, but to show the effect of increasing CO_2 concentrations. Therefore, the model is adjusted to the known surface temperature of the reference year 1990. The results are shown in Fig. 13. To get 15 °C as the Earth temperature in 1990, the cloud emissivity was adjusted to 0.778.

3. Influence of CO_2 concentration

Based on the previously presented equations, the dependency of the Earth's surface temperature on the carbon dioxide concentration was calculated. Fig. 14 shows how the temperature of the Earth increases with the CO_2 concentration. Without CO_2, the temperature of the Earth would be 4 K colder.

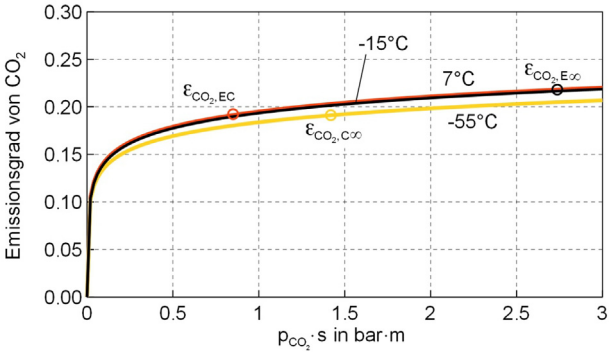

Fig. 11. Emissivity of carbon dioxide in the atmosphere.

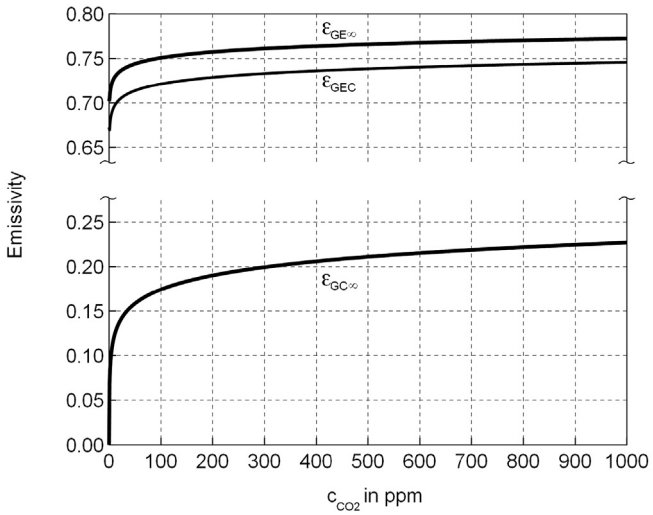

Fig. 12. Variation of emissivity as a function of CO_2 concentration.

Up to 350 ppm, the temperature of the Earth increases significantly with the CO_2 concentration. It is shown that for a doubling of the carbon dioxide concentration from 355 ppm to 710 ppm, the temperature increases from 15 °C to 15.41 °C. Therefore the climate sensitivity for this model is 0.41 °C.

For the current climate analysis, it is of great interest to know how strongly CO_2 influences the climate and to what extent the temperature of the Earth increases with an increase in the CO_2 content. The measure of how sensitively the temperature reacts to a change in the carbon dioxide content is called the climate sensitivity. Climate sensitivity is the change of the average global ground level temperature caused by a doubling of the CO_2 concentration. It is here assumed that the system is at equilibrium before and after the change. There are several possibilities for the assessment of how the climate system reacts to a doubling of the carbon dioxide concentration. The climate sensitivity can be determined with the help of models using simple coupling mechanisms, assumptions and given conditions or by simulation. Thereby, predictions for the future temperature increases can be predicted. In Table 3, increases in temperature with a doubling of the CO_2 concentration are compared from past and recent studies.

Furthermore, it is possible to analyze the climate in the past. How did the system react to a change in the carbon dioxide concentration? Using the measured data and the state of knowledge of past climate fluctuations, the influence of CO_2 concentration on the climate during a certain period of time can be calculated. Lorius et al. [28] established a study regarding this issue.

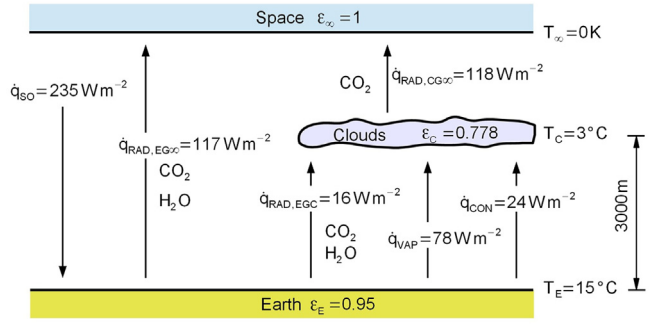

Fig. 13. Heat fluxes in the atmosphere.

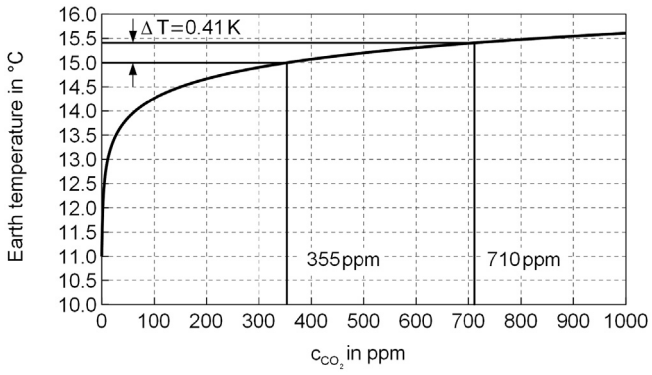

Fig. 14. Increase of Earth temperature with CO_2 concentration.

The values from ice drilling of polar ice plates show a correlation between the change in the Ice Age temperature and carbon dioxide. From these investigations, it is known that the climate sensitivity of the last ca. one million years was 3–4 °C. Royer et al. [29] examined how carbon dioxide has influenced the temperature over the last 420 million years. A climate sensitivity in the area of 1.5–6.2 °C was revealed (best assessment 2.8 °C). But how well can the climate of the past be reconstructed?

A group of climate scientists from the Potsdam Institute for Climate Tracking Analysis (PIK) [30] has designed a new study, in which the newly obtained data sets from the climate model are combined with the Ice Age climate. The climate sensitivity was determined for 1000 different models with various influence parameters. These were compared with the data from the Ice Age. The calculated results could not be reconciled with the data from the last ice age for climate sensitivities less than 1.2 °C or higher than 4.3 °C.

More recent papers related to this work can be found in the literature. By using a one-dimensional radiative heat transfer model for atmosphere Liu et al. [31] demonstrated how the CO_2 influenced the greenhouse effect. The results show that temperature in troposphere rises 0.453 °C if doubling of CO_2 concentration. With the atmosphere-ocean general circulation models Meinshausen et al. [32] identified an average value of 2.88 °C for the climate sensitivity.

Under consideration of all mentioned studies, the temperatures for doubling of CO_2 concentration range from 0.4 °C to even 9.2 °C. The temperature increase calculated with the simple model presented in this paper lies in the lower range. The climate sensitivity from 5 °C up to 9 °C should be considered as critical.

4. Conclusions

With a simple one dimensional mathematical model, the mechanism of global warming is explained. The fundamental

Table 3
Climate sensitivity.

Climate sensitivity ΔT in °C	Source
0.6	Lorenz [16],
2.9	Manabe and Wetherald [19],
1.9	Augustsson and Ramanathan [20],
0.4	Idso [21],
2.4–5.4	Murphy [22],
2.4–9.2	Forest et al. [23],
2–4.5	IPCC [24],
0.6–1.7	Schwartz [25],
0.5	Lindzen [26],
0.6	Harde [27],

physical principles generally used in the engineering field are used to calculate the heat fluxes due to radiation with an atmosphere containing CO_2 and H_2O. The emissivity of the clouds acts as an adjustment parameter.

The temperature of the Earth increases significantly with the CO_2 concentration. While doubling the CO_2 concentration, the temperature of the Earth increases by 0.4 K. This increase lies in the lower range of the complex models of different global climate research institutes. Since 1860, the Earth's temperature has already risen due to anthropogenic CO_2 emissions by 0.4 K. The measured increase of about 1 K is attributed to side effects caused by the CO_2 related temperature increase. Therefore, a temperature increase of more than 0.4 K is predicted for the future.

References

[1] S.K. Foong, An accurate analytical solution of a zero-dimensional greenhouse model for global warming, Eur. J. Phys. 27 (2006) 933–942.
[2] R.S. Knox, Physical aspects of the greenhouse effect and global warming, Am. J. Phys. 67 (1999) 1227–1238.
[3] J.R. Barker, M.H. Ross, An introduction to global warming, Am. J. Phys. 67 (1999) 1216–1226.
[4] P. Onorato, P. Mascheretti, A. De Ambrosis, 'Home made' model to study greenhouse effect and global warming, Eur. J. Phys. 32 (2011) 363–376.
[5] J.T. Kiehl, K.E. Trenberth, Earth's annual global mean energy Budget, Bull, Am. Meteorological Soc. 78 (1997) 197–208.
[6] U.S. Department of Commerce, NatioNal Oceanic and Atmospheric Administration, Earth System Research Laboratory, Global Monitoring Division, http://www.esrl.noaa.gov/gmd, (last access 20.10.14).
[7] W.-C. Wang, W.B. Rossow, M.-S. Yao, M. Wolfson, Climate sensitivity of a one-dimensional radiative-convective model with cloud feedback, J. Atmos. Sci. 38 (1981) 1167–1178.
[8] K.N. Liou, An Introduction to Atmospheric Radiation, in: International Geophysics Series, second ed., vol. 84, Academic Press, San Diego, 2002.
[9] M.F. Modest, Radiative Heat Transfer, McGraw-Hill Series in Mechanical Engineering, 1993.
[10] E. Specht, Heat and Mass Transfer in Thermal Process Engineering, Vulkan Verlag Essen, 2014 (in German).
[11] A.F. Mills, Basic Heat and Mass Transfer, Prentice Hall, Upper Saddle River, New Jersey, 1999.
[12] K.Ya Kondratyev, Radiation in the Atmosphere, Int. Geophys. Ser. 12 (1969) 124–129.
[13] W. Roedel, Physics of the Environment. The Atmosphere, Springer Publisher, Berlin, Heidelberg, New York, Germany, 1992 (in German).
[14] W. Weischet, W. Endlicher, Introduction to the General Climatology, Gebrüder Borntraeger Verlagsbuchhandlung, Stuttgart, Germany, 2008 (in German).
[15] N. Lallemant, A. Sayre, R. Weber, Evaluation of emissivity correlations for H_2O-CO_2-N_2/Air mixtures and coupling with solution methods of the radiative transfer equation, Prog. Energy Combust. Sci. 22 (1996) 543–574.
[16] N. Lorenz, Simplified One-dimensional Model for Calculating Global Warming through Anthropogenic CO_2, Dissertation, Otto von Guericke University Magdeburg, 2012 (in German).
[17] I.H. Farag, T.A. Allam, Water vapor standard emissivity by mixed gray-gases model, Chem. Eng. Commun. 15 (1982) 215–226.
[18] O. Marin, R.O. Buckius, A simplified wide band model for the cumulative distribution function for carbon dioxide, Int. J. Heat. Mass Transf. 41 (1998) 3881–3897.
[19] S. Manabe, R.T. Wetherald, The effects of doubling CO_2 concentration on the climate of a general circulation model, J. Atmos. Sci. 32 (1975) 3–15.
[20] T. Augustsson, V.A. Ramanathan, A radiative-convective model study of the CO_2 climate problem, J. Atmos. Sci. 34 (1977) 448–451.
[21] S.B. Idso, CO_2-incluced global warming: a skeptic's view of potential climate change, Clim. Res. 10 (1998) 69–82.
[22] J.M. Murphy, D.M.H. Sexton, D.N. Barnett, G.S. Jones, M.J. Webb, M. Collins, D.A. Stainforth, Quantification of modeling uncertainties in a large ensemble of climate change simulations, Nature 430 (2004) 768–772.
[23] C.E. Forest, P.H. Stone, A.P. Sokolov, Estimated PDFs of climate system properties including natural and anthropogenic forcings, Geophys. Res. Lett. 33 (2005).
[24] Intergovernmental Panel on Climate Change (IPCC), Fourth Assessment Report – Working Group I Report on "The Physical Science Basis", Geneva, Switzerland, 2007.
[25] S.E. Schwartz, Heat capacity, time constant and sensitivity of earth's climate system, J. Geophys. Res. 112 (2007).
[26] R.S. Lindzen, Y.S. Choi, On the determination of climate feedbacks from ERBE data, Geophys. Res. Lett. 36 (2009).
[27] Hermann Harde, Advanced two-layer climatemodel for the assessment of global warming by CO_2, Open J. Atmos. Clim. Change 1 (3) (2014) 1–50.
[28] C. Lorius, et al., The ice-core record, climate sensitivity and future greenhouse warming, Nature 347 (1990) 139–145.
[29] D.L. Royer, R.A. Berner, J. Park, Climate sensitivity constrained by CO_2 concentrations over the past 420 million years, Nature 446 (2007), 530–523.
[30] T. Schneider von Deimling, H. Held, A. Ganopolski, S. Rahmstorf, Climate sensitivity estimated from ensemble simulations of glacial climate, Clim. Dynam 27 (2006) 149–163.
[31] B. Liu, Y. Shuai, H.P. Tan, One dimensional radiative heat transfer analysis of atmosphere greenhouse effects, J. Eng. Thermophys. 32 (6) (2011) 1012–1014.
[32] M. Meinshausen, S.C.B. Raper, T.M.L. Wigley, Emulating coupled atmosphere-ocean and carbon cycle models with a simpler model, MAGICC6-Part 1: model description and calibration, Atmos. Chem. Phys. 11 (4) (2011) 1417–1456.